MATHS

NUMBER SYSTEM

The system used for naming or representing numbers, as the decimal system or the binary system. It is also called numeral system.

In mathematics, the phrase *number system* refers to any collection of objects that has the following properties:

- there is a rule for how to add two objects together,
- there is a rule for how to multiply two objects together,
- the rules for addition and multiplication satisfy the familiar properties of arithmetic, such as
  - *commutativity* (order doesn't matter)
  - *associativity* (in a sum of three of more terms, it doesn't matter which two you add first, and likewise for products)
  - *distributivity* ($a(b+c) = ab + ac$)
- All objects have corresponding *negatives* (from which we can define subtraction as the addition of one object to the negative of another), and some objects have reciprocals (from which we can define division as the multiplication of one object by the reciprocal of another). The properties of subtraction and division

follow automatically from the corresponding properties of addition and multiplication, so we need not mention them explicitly.

Types of Number Systems

The most common number systems are the following:

The Natural Numbers- These are the counting numbers 1, 2, 3, ... that are possible answers to the question "how many?" They are abstract concepts that describe sizes of sets.

The Integers- These are abstract concepts that describe, not sizes of sets, but the relative sizes of two sets. They are the possible answers to the question "how many more does A have than B has?" They include both positive numbers (meaning A has more than B) and negative numbers (meaning B has more than A).

The Rational Numbers- These are abstract concepts that describe ratios of sizes of sets. They do not model sizes of sets the way that natural numbers do. If you say "I ate 3/4 of a pie", you are not saying that the set of things you ate had 3/4 elements. Instead, you are expressing a ratio of two integer quantities: 3, the number of pie-quarters that you ate, and 4, the number of pie-quarters that make up a whole pie.

The Real Numbers- These are abstract concepts that describe measurements of continuous quantities, such as length, weight, quantity of fluid, etc. (Don't let the word "real" fool you; the real numbers are no more "real" in the ordinary English sense of the word than are any other kind of numbers.)

The Complex Numbers- These are pairs of real numbers, with pairs of the form (x , 0) behaving the same as ordinary real numbers x, but with other pairs (whose second entry is non-zero) behaving differently. The rule for multiplication is

$$(a, b) \text{ times } (c, d) = (ac - bd , ad + bc)$$

which means that the pair (0,1), when squared, gives (-1,0) which in this context is considered to be the same as the real number -1 (since a complex number of the form (x,0) is indistinguishable by its arithmetic properties from the real number x, we can consider it to be describing the same underlying concept, much as we consider a fraction of the form x/1 to be describing the same underlying concept as the integer x).

TRICKS:

1. Method to multiply 2-digit number.

(i) AB × CD = AC / AD + BC / BD

35 × 47 = 12 / 21 + 20 / 35 = 12 / 41 / 35 = 1645

(ii) AB × AC = A2 / A (B + C) / BC

74 × 76 = 72 / 7(4 + 6) / 4 × 6 = 49 / 70 / 24

= 49 / 70 / 24 = 5624

(iii) AB × CC = AC / (A + B) C / BC

35 × 44 = 3 × 4 / (3 + 5) × 4 / 5 × 4 = 12 / 32 / 20

= 12 / 32 / 20 = 1540

2. Method to multiply 3-digit no.

ABC × DEF = AD / AE + BD / AF + BE + CD / BF + CE / CF

456 × 234 = 4 × 2 / 4 × 3 + 5 × 2 / 4 × 4 + 5 × 3 + 6 × 2 / 5 × 4 + 6 × 3 / 6 × 4

= 8 / 12 + 10 / 16 + 15 + 12 / 20 + 18 / 24

= 8 / 22 /43 / 38 / 24

= 106704

3. If in a series all number contains repeating 7.

To find their sum, we start from the left multiply 7 by 1, 2, 3, 4, 5 & 6. Look at the example below.

777777 + 77777 + 7777 + 777 + 77 + 7 = ?

= 7 × 1 / 7 × 2 / 7 × 3 / 7 × 4 / 7 × 5 / 7 × 6

= 7 / 14 / 21 / 28 / 35 / 42 = 864192

4. To find the sum of those number in which one number is repeated after decimal.

First write the number in either increasing or decreasing order then find the sum by using the below method.

0.5555 + 0.555 + 0.55 + 0.5 = ?

= 5 × 4 / 5 × 3 / 5 × 2 / 5 × 1

= 20 / 15 / 10 / 5 = 2.1605

5. Those numbers whose all digits are 3.

Those number in which all digits are number is 3 two or more than 2 times repeated, to find the square of these number, we repeat 1 and 8 by (n – 1) time. Where n ® Number of times 3 repeated.

$(33)^2$ = 1089

$(333)^2$ = 110889

$(3333)^2$ = 11108889

6. Those number whose all digits are 9.

$(99)^2$ = 9801

$(999)^2$ = 998001

$(9999)^2$ = 99980001

$(99999)^2$ = 9999800001

7. Those number whose all digits are 1.

A number whose one's, ten's, hundred's digit is 1 i.e., 11, 111, 1111,.... In this we count number of digits. We write 1, 2, 3, ... in their square the digit in the number, then write in decreasing order up to 1.

$(11)^2$ = 121

$(111)^2$ = 12321

$(1111)^2$ = 1234321

8. Some properties of square and square root.

- Complete square of a no. is possible if its last digit is 0, 1, 4, 5, 6 & 9. If last digit of a no. is 2, 3, 7, 8 then complete square root of this no. is not possible.
- If last digit of a no. is 1, then last digit of its complete square root is either 1 or 9.
- If last digit of a no. is 4, then last digit of its complete square root is either 2 or 8.
- If last digit of a no. is 5 or 0, then last digit of its complete square root is either 5 or 0.
- If last digit of a no. is 6, then last digit of its complete square root is either 4 or 6.
- If last digit of a no. is 9, then last digit of its complete square root is either 3 or 7.

9. Prime Number.

- Find the approx square root of given no. Divide the given no. by the prime no. less than approx square root of no. If given no. is not divisible by any of these prime no. then the no. is prime otherwise not.

For example : Is 359 is a prime number or not?

Approx sq. root = 19

Prime no. < 19 are 2, 3, 5, 7, 11, 13, 17

359 is not divisible by any of these prime nos. So 359 is a prime no.

For example: Is 2^{5001} + 1 is prime or not?

$$\frac{2^{5001}+1}{2+1} = \text{Reminder -> 0}$$

Therefore , 2^5001+1 is not prime.

- There are 15 prime no. from 1 to 50.
- There are 25 prime no. from 1 to 100.
- There are 168 prime no. from 1 to 1000.

10. If a no. is in the form of $x^n + a^n$.

Then the number is divisible by (x + a), if n is odd.

11. If a no. is in the form of $\frac{x^n}{(x - 1)}$.

Then the remainder is always 1.

12. If a no. is in the form of x^n ÷ (x + 1).

- If n is even, then remainder is 1.
- If n is odd, then remainder is x.

13. Number of divisors:

- If N is any no. and

  N = $a^n \times b^m \times c^p \times$ .... where a, b, c are prime no.

  No. of divisors of N = (n + 1) (m + 1) (p + 1) ....

  <u>*For example*</u> : Find the no. of divisors of 90000.

  N = 90000 = $2^2 \times 3^2 \times 5^2 \times 10^2$

  $= 2^2 \times 3^2 \times 5^2 \times (2 \times 5)^2$

  $= 2^4 \times 3^2 \times 5^4$

  So, the no. of divisors = (4 + 1) (2 + 1) (4 + 1) = 75

- N = $a^n \times b^m \times c^p$, where a, b, c are prime

  Then set of co-prime factors of N

  = [(n + 1) (m + 1) (p + 1) – 1 + nm + mp + pn + 3mnp]

- If N = $a^n \times b^m \times c^p \ldots$ where a, b & c are prime no.

  Then sum of the divisors = $\frac{(a^{n+1}-1)(b^{m+1}-1)(c^{p+1}-1)}{(a-1)(b-1)(c-1)}$

15. To find the last digit or digit at the unit's place of a^n.

If the last digit or digit at the unit's place of a is 1, 5 or 6, whatever be the value of n, it will have the same digit at unit's place, i.e.

$(\ldots\ldots 1)^n$ = (.....1)

$(\ldots\ldots 5)^n$ = (.....5)

$(\ldots\ldots 6)^n$ = (.....6)

16. To find the sum of a series.

- Sum of n natural number = $\frac{n(n+1)}{2}$
- Sum of n even number = (n) (n + 1)
- Sum of n odd number = n^2

17. To find the square of a series.

- Sum of sq. of first n natural no. = $\frac{n(n+1)(2n+1)}{6}$
- Sum of sq. of first n odd natural no. = $\frac{n(4n^2-1)}{3}$
- Sum of sq. of first n even natural no. = $\frac{2n(n+1)(2n+1)}{3}$

18. To find the cube of a series.

- Sum of cube of first n natural no. = $\frac{n^2(n+1)^2}{4}$
- Sum of cube of first n even natural no. = $2n^2(n+1)^2$
- Sum of cube of first n odd natural no. = $n^2(2n^2-1)$

19. Divisiblity.

- $x^n - y^n$ is divisible by (x + y)

  When n is even.

- $x^n - y^n$ is divisible by (x – y)

  When n is either odd or even.

20. For any integer n.

- $(n^3 - n)$ is divisible by 3
- $(n^5 - n)$ is divisible by 5
- $(n^{11} - n)$ is divisible by 11
- $(n^{13} - n)$ is divisible by 13

21. Some articles related to Divisibility.

- A no. of 3-digits which is formed by repeating a digit 3-times, then this no. is divisible by 3 and 37.

  e.g., 111, 222, 333, ...

- A no. of 6-digit which is formed by repeating a digit 6-times then this no. is divisible by 3, 7, 11, 13 and 37.

  e.g., 111111, 222222, 333333, 444444, ...

22. Divisible by 7.

We use osculator (– 2) for divisibility test.

99995 : 9999 – 2 × 5 = 9989

9989 : 998 – 2 × 9 = 980

980 : 98 – 2 × 0 = 98

Now 98 is divisible by 7. So 99995 is also divisible by 7.

23. Divisible by 11.

In a number, if difference of sum of digit at even places and sum of digit at odd places is either 0 or multiple of 11, then no. is divisible by 11.

For example : 12342÷11

Sum of even place digit = 2 + 4 = 6

Sum of odd place digit = 1 + 3 + 2 = 6

Difference = 6 – 6 = 0

Therefore, 12342 is divisible by 11.

24. Divisible by 13.

We use (+ 4) as osculator.

For example : 876538÷ 13

876538: 8 × 4 + 3 = 35

5 × 4 + 3 + 5 = 28

8 × 4 + 2 + 6 = 40

0 × 4 + 4 + 7 = 11

1 × 4 + 1 + 8 = 13

13 is divisible by 13.

Therefore, 876538 is also divisible by 13.

25. Divisible by 17.

We use (– 5) as osculator.

For example : 294678: 29467 – 5 × 8 = 29427

27427: 2942 – 5 × 7 = 2907

2907: 290 – 5 × 7 = 255

255: 25 – 5 × 5 = 0

Therefore, 294678 is completely divisible by 17.

26. Divisible by 19.

We use (+ 2) as osculator.

For example : 149264: 4 × 2 + 6 = 14

4 × 2 + 1 + 2 = 11

1 × 2 + 1 + 9 = 12

2 × 2 + 1 + 4 = 9

9 × 2 + 1 = 19

19 is divisible by 19.

Therefore, 149264 is divisible by 19.

27. HCF (Highest Common factor).

There are two methods to find the HCF:

(a) Factor method (b) Division method

- For two no. a and b if a < b, then HCF of a and b is always less than or equal to a .
- The greatest number by which x, y and z completely divisible is the HCF of x, y and z.
- The greatest number by which x, y, z divisible and gives the remainder a, b and c is the HCF of (x –a), (y–b) and (z–c).
- The greatest number by which x, y and z divisible and gives same remainder in each case, that number is HCF of (x–y), (y–z) and (z–x).
- H.C.F of $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f} = \frac{H.C.F\ of\ (a,c,e)}{L.C.M\ of\ (b,d,f)}$

28. LCM (Least Common Multiple).

There are two methods to find the LCM–

(a) Factor method (b) Division method

- For two numbers a and b if a < b, then L.C.M. of a and b is more than or equal to b.
- If ratio between two numbers is a:b and their H.C.F. is x, then their L.C.M. = abx.
- If ratio between two numbers is a:b and their L.C.M. is x, then their H.C.F. $= \frac{x}{ab}$
- The smallest number which is divisible by x, y and z is L.C.M. of x, y and z.

- The smallest number which is divided by x, y and z give remainder a, b and c, but (x – a) = (y – b) = (z – c) = k, then number is (L.C.M. of (x, y and z) – k).
- The smallest number which is divided by x, y and z give remainder k in each case, then number is (L.C.M. of x, y and z) + k.
- L.C.M. of $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ $= \frac{L.C.M\ of\ (a,c,e)}{H.C.F\ of\ (b,d,f)}$
- For two numbers a and b

  LCM × HCF = a × b.

- If a is the H.C.F. of each pair from n numbers and L is L.C.M., then product of n numbers $= a^{n-1}L$.

ALGEBRA

A branch of *mathematics* in which symbols, usually letters of the alphabet, represent numbers or members of a specified set and are used to represent quantities and to express general relationships that hold for all members of the set.

Algebra Identities:

- $(a + b)^2 + (a - b)^2 = 2\,(a^2 + b^2)$
- $(a + b)^2 - (a - b)^2 = 4ab$
- $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$
- $a^4 + a^2 + 1 = (a^2 + a + 1)\,(a^2 - a + 1)$
- $If\ a + b + c = 0, then\ a^3 + b^3 + c^3 = 3abc$
- $\frac{(a+b)^2-(a-b)^2}{ab} = 4$

- $\frac{(a+b)^2+(a-b)^2}{a^2+b^2} = 2$

An algebraic expression is a mathematical phrase that can contain ordinary numbers, variables (like x or y) and operators (like add, subtract, multiply and divide). Here are some algebraic expressions: a + 1, a – b, 3x.

TRICKS:

1.If a1x + b1y = c1 and a2x + b2y = c2.

- If $\frac{a1}{a2} \neq \frac{b1}{b2}$, one solution.
- If $\frac{a1}{a2} = \frac{b1}{b2} = \frac{c1}{c2}$,Infinite many solutions.
- If $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$, No solution

2. If α and β are roots of ax^2 + bx + c = 0.

$\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are roots of cx^2 + bx + a = 0

3. If β and α are roots of ax^2+ bx + c = 0.

- One root is zero if c = 0.
- Both roots zero if b = 0 and c = 0.
- Roots are reciprocal to each other, if c = a.
- If both roots α and β are positive, then sign of a and b are opposite and sign of c and a are same.
- If both roots α and β are negative, then sign of a, b and c are same.

$$(\alpha + \beta) = \frac{-b}{a} , \alpha\beta = \frac{c}{a}$$

$$(\alpha - \beta) = \sqrt{}((\alpha + \beta)^2 - 4\alpha\beta)$$

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$$

4. Arithmetic Progression.

- If a, a + d, a + 2d, ... are in A.P., then nth term of A.P.

  A(n) = a + (n – 1)d

  and Sum of n terms of this A.P is

  S(n) = $\frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$

  where, a = first term d = common difference l= last term

- A.M.(Arithmetic mean) = $\frac{a+b}{2}$

5. Geometric Progression.

- G.P. - a, ar, ar^2, ar^3, ar^4……...

  Then, nth term of G.P. a(n) $=\ ar^{n-1}$

  S (n) = $\frac{a(r^n-1)}{r-1}$ where r>1

  = $\frac{a(1-r^n)}{1-r}$ where r<1

  S (∞) = $1 - r$

  r = common ratio and a = first term

- G.M. =$\sqrt{ab}$

6. If a, b, c are in H.P. and $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

n^{th} term of H.M. = $\frac{1}{n^{th} term\ of\ A.P.}$

H.M. = $\frac{2ab}{a+b}$

<u>Note</u>: Relation between A.M., G.M. and H.M.

A.M. × H.M. = $G.M.^2$ & A.M. > G.M. > H.M.

Where, A.M. is Arithmetic Mean ,G.M. is Geometric Mean,

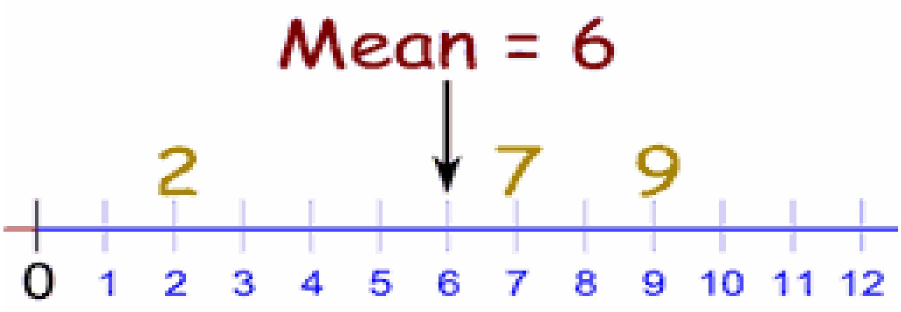
H.M. is Harmonic Mean

AVERAGE

A number expressing the central or typical value in a set of data, in particular the mode, median, or (most commonly) the mean, which is calculated by dividing the sum of the values in the set by their number.

Example: what is the average of 2, 7 and 9?

Add the numbers: 2 + 7 + 9 = 18



Divide by how many numbers (i.e. we added 3 numbers):

18 ÷ 3 = 6, So the average is 6.

TRICKS:

1. Average of n numbers.

- Average of first n natural no. = $\frac{n+1}{2}$
- Average of first n even no. = (n + 1)
- Average of first n odd no. = n

2. Average of sum of square of n numbers.

- Average of sum of square of first n natural no. = $\frac{(n+1)(2n+1)}{6}$
- Average of sum of square of first n even no. = $\frac{2(n+1)(2n+1)}{3}$
- Average of sum of square of first odd no. = $\frac{4n^2-1}{3}$

3. Average of cube of n numbers.

- Average of cube of first n natural no. = $\frac{n(n+1)^2}{4}$
- Average of cube of first n even natural no. = $2n(n+1)^2$
- Average of cube of first n odd natural no. = $n(2n^2 - 1)$

4. Average of first n multiple of m.

$$x = m(n+1)/2$$

5. (i) If average of some observations is x and a is added in each observations, then new average is (x + a).

(ii) If average of some observations is x and a is subtracted in each observations, then new average is (x – a).

(iii) If average of some observations is x and each observations multiply by a, then new average is ax.

(iv) If average of some observations is x and each observations is divided by a, then new average is $\frac{x}{a}$.

(v) If average of n1 is A1, & average of n2 is A2, then Average of (n1 + n2) is 1 1 2 2 12 n A n A nn + + and

Average of (n1 – n2) is 1 1 2 2 12 n A n A nn

6. When a person is included or excluded the group.

Age/Weight of that person = No. of persons in group × (Increase / Decrease) in average ± New average.

For example: In a class average age of 15 students is 18 yrs. When the age of teacher is included their average increased by 2 yrs, then find the age of teacher.

Sol. Age of teacher = 15 × 2 + (18 + 2) = 30 + 20 = 50 yrs.

7. If a person travels two equal distances at a speed of x km/h and y km/h.

Average speed = $\frac{2xy}{x+y}$ km/h

8. If a person travels three equal distances at a speed of x km/h, y km/h and z km/h.

Average speed = $\frac{3xyz}{xy+yz+zx}$ km/h.

RATIO AND PROPORTION

A proportion is a name we give to a statement that two ratios are equal. It can be written in two ways: two equal fractions, or, using a colon, a:b = c:d. It can be written in two ways:

- two equal fractions, $\frac{a}{b} = \frac{c}{d}$
- using a colon,   a:b = c:d

When two ratios are equal, then the cross products of the ratios are equal. That is, for the proportion

$$a : b = c : d,\ \ a \times d = b \times c$$

TRICKS:

1. If ratio is written as A:B , it is said to be a linear form and in case it is written as A/B, it is said to be fractional form.
2. Ratio does not have any unit. It is mere number.
3. The equality of two ratios is known as proportion i.e. a/b = c/d
   If $\frac{a}{b} = \frac{c}{d}$, then it is also equal to $\frac{a+c}{b+d}$.

Invertendo : If a/b = c/d , then b/a = d/c

Alterendo : If a/b = c/d , then a/c = b/d

Componendo : If a/b = c/d , then a+b/b = c+d/d

Dividendo : If a/b = c/d , then a-b/b = c-d/d

Componendo and Dividendo : If a/b = c/d , then a+b/a-b = c+d/c-d

4. If a/b = b/c = c/d =...... so on, then a, b, c, d... are in G.P.

   Proof: Let a/b = b/c = c/d =k

   c= dk, b= ck, a= bk

Therefore, b= dk^2 and a = dk^2. All are in G.P.

5. If a>b and same positive number is added to each term, then ratio decreases.
For example: a/b = 4/3 = 1.3
If 2 is added to each term, then
a/b = 4+2/3+2 = 6/5 = 1.2 (ratio decreases)

6. If a<b and same positive number is added to each term, then ratio increases.
For example: a/b = 3/4 = 0.7
If 2 is added to each term, then
a/b = 3+2/4+2 = 5/6 = 0.8 (ratio increases)

7. If we multiply or divide any number, there will be no effect on ratio.

8. Let a:b is a ratio

$a^2 : b^2$ is duplicate ratio of a:b

$a^3 : b^3$ is triplicate ratio of a:b

$a^{\frac{1}{2}} : b^{\frac{1}{2}}$ is sub-duplicate ratio of a:b

$a^{\frac{1}{3}} : b^{\frac{1}{3}}$ is sub-triplicate ratio of a:b

9. Proportions i.e. a:b = c:d

a and d are known to be extremes

b and c are known to be means.

SIMPLE AND COMPOUND INTEREST

If P = Principal, R = Rate per annum, T = Time in years(or n year), SI = Simple interest, A = Amount

- SI = $\frac{P \times R \times T}{100}$
- A = P + SI = $P\left[1+\frac{RT}{100}\right]$
- A = $\left[1+\frac{R}{100}\right]^{n}$, interest payable annually.
- A = $P\left[1+\frac{R'}{100}\right]^{n'}$, interest payable half-yearly

  $$R' = \frac{R}{2}\ and\ n' = 2n$$

- A = $P\left[1+\frac{R}{400}\right]^{4n}$, interest payable quarterly.
- $\left[1+\frac{R}{400}\right]$ is the yearly growth factor.
- $\left[1-\frac{R}{400}\right]$ is the yearly decay factor or depreciation factor.
- CI = Amount – Principal = $P\left[\left(1+\frac{R}{100}\right)^{n}-1\right]$
- When Rates are different for different years, say R1, R2, R3% for 1st, 2nd & 3rd years respectively, then

  Amount =$P\left[1+\frac{R1}{100}\right]\left[1+\frac{R2}{100}\right]\left[1+\frac{R3}{100}\right]$

PROBLEMS ON TRAIN

- **To convert km/hr to m/sec.**

We know, km/hr is bigger value and m/sec is smaller so we divide by a value which has greater denominator than numerator. so to convert km/hr into m/sec we multiply by 5÷18, where 18 (denominator) is greater than 5 (numerator).

$$X\ km/hr = X \times \frac{5}{18}\ m/sec$$

- **To convert m/sec to km/hr**

Similarly we do the reverse process

$$X\ m/sec = X \times \frac{18}{5}\ km/hr$$

- A train running on S speed having L length and it is crossing a standing man then total time taken by train to cross the man is,

T (Time Taken) = L (Length of train) / S (Speed of train)

Note: Time taken by train (L length) to cross the signal/standing man is equal to the time taken by the train to cover L distance.

- **Relative Speed of train in Same Direction**

If two trains/Objects are moving in the same direction at v1 m/s and v2 m/s respectively where v1>v2, then their relative speed is (v1−v2) m/s.

- **Relative Speed of train in Opposite Direction**

If two trains/Objects are moving in the Opposite direction at v1 m/s and v2 m/s respectively, then their relative speed is (v1+v2) m/s.

- **Total Distance**

If train (length L) is crossing a standing man/pole then total distance traveled will be **L**.

If a train(length L1) is crossing a bridge/station(length L2) then total distance traveled will be **L1 + L2.**

If one train (length L1) is crossing another train (length L2) in same direction then total distance traveled will be **L1 + L2.**

If one train (length L1) is crossing another train (length L2) in opposite direction then total distance traveled will be **L1 + L2.**

- **Time taken by two trains to cross each other(opposite direction)**

If two trains of length a metres and b metres are moving in opposite directions at u m/s and v m/s, then time taken by the trains to cross each other = (a + b)/(u + v) sec.

- **Time taken by two trains to cross each other (same direction)**

If two trains of length a metres and b metres are moving in the same direction at u m / s and v m / s, then the time taken by the faster train to cross the slower train = (a + b)/(u - v) sec.

- **Crossing trains each other**

If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then

(A's speed):(B's speed) = ($\sqrt{b1}$: $\sqrt{a1}$)

GEOMETRY

- Sum of all the exterior angle of a polygon = 360°
- Each exterior angle of a regular polygon = $\frac{360^\circ}{n}$
- Sum of all the interior angles of a polygon = (n – 2) × 180°
- Each interior angle of a regular polygon = $\frac{n-2}{n} \times 180^\circ$
- No. of diagonals of a polygon = $\frac{n(n-3)}{2}$, n(no. of sides)
- The ratio of sides a polygon to the diagonals of a polygon is 2 : (n – 3)
- Ratio of interior angle to exterior angle of a regular polygon is (n – 2) : 2

Properties of triangle:

- When one side is extended in any direction, an angle is formed with another side. This is called the exterior angle. There are six exterior angles of a triangle.
- Interior angle + corresponding exterior angle = 180°.
- An exterior angle = Sum of the other two interior opposite angles.
- Sum of the lengths of any two sides is greater than the length of third side.
- Difference of any two sides is less than the third side. Side opposite to the greatest angle is greatest and vice versa.
- A triangle must have at least two acute angles.
- Triangles on equal bases and between the same parallels have equal areas.
- If a, b, c denote the sides of a triangle then
  1. if c2 < a2 + b2, Triangle is acute angled.
  2. if c2 = a2 + b2, Triangle is right angled.
  3. if c2 > a2 + b2, Triangle is obtuse angled.

Properties of circle:

- Only one circle can pass through three given points.
- There is one and only one tangent to the circle passing through any point on the circle.
- From any exterior point of the circle, two tangents can be drawn on to the circle.
- The lengths of two tangents segment from the exterior point to the circle are equal.
- The tangent at any point of a circle and the radius through the point are perpendicular to each other.
- When two circles touch each other, their centres & the point of contact are collinear.
- If two circles touch externally, distance between centres = sum of radii.
- If two circles touch internally, distance between centres = difference of radii
- Circles with same centre and different radii are concentric circles.
- Points lying on the same circle are called concyclic points.
- Measure of an arc means measure of central angle.
  m (minor arc) + m (major arc) = 360°.
- Angle in a semicircle is a right angle.
- Only one circle can pass through three given

Units of Measurement of Area and Volume

The inter-relationships between various units of measurement of length, area and volume are listed below for ready reference:

Length
1 Centimetre (cm) = 10 milimetre (mm)
1 Decimetre (dm) = 10 centimetre
1 Metre (m) = 10 dm = 100 cm = 1000 mm
1 Decametre (dam) = 10 m = 1000 cm
1 Hectometre (hm) = 10 dam = 100 m
1 Kilometre (km) = 1000 m = 100 dam = 10 hm
1 Myriametre = 10 kilometre

Area
1 cm2 = 1 cm × 1 cm = 10 mm × 10 mm = 100 mm2
1 dm2 = 1 dm × 1 dm = 10 cm × 10 cm = 100 cm2
1 m2 = 1 m × 1 m = 10 dm × 10 dm = 100 dm2
1 dam2 or 1 are = 1 dam × 1dam = 10 m × 10 m = 100 m2
1 hm2 = 1 hectare = 1 hm × 1 hm =100 m × 10000m2=100 dm2
1 km2 = 1 km × 1 km = 10 hm × 10 hm=100 hm2 or 100 hectare

Volume
1 cm3 = 1 ml = 1 cm × 1 cm × 1 cm = 10 mm × 10 mm × 10 mm = 1000 mm3
1 litre = 1000 ml = 1000 cm3
1 m3 = 1 m×1 m×1m = 100 cm × 100 cm × 100 cm = 106 cm3 = 1000 litre = 1 kilometre
1 dm3 = 1000 cm3, 1m3 = 1000 dm3, 1 km3 = 109 m3